BACKPAPER: ALGEBRAIC GEOMETRY

Date: **May 2015**

A **ring** would mean a **commutative ring with identity**.

(1) (20 points) Let X be a reduced noetherian scheme. Show that there exist at least one and at most finitely points $x_1, \ldots, x_n$ in X such that the stalk(s) $\mathcal{O}_{X,x_i}$ are fields.

(2) (20 points) Let S be a scheme and let X and Y be a S-scheme. Prove or Disprove
   (a) If S, X and Y are irreducible then $X \times_S Y$ is irreducble.
   (b) If $X \to S$ and $Y \to S$ are separated then $X \times_S Y \to S$ is separated.

(3) (5+15=20 points) When is a morphism of schemes called proper? Show that finite morphisms are proper.

(4) (5+15=20 points) Let X be a noetherian connected nonsingular scheme. Show that X is an integral scheme.

(5) (20 points) Let X be the projective plane curve over a field k given by the irreducible homogeneous polynomial $zy^3 - x^3(x - z)$. Show that the normalization of X is isomorphic to $\mathbb{P}^1$.